A verified runtime for a verified theorem prover

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ITP 2011

Two projects meet

Jared Davis

A self-verifying theorem prover

Milawa

Magnus Myreen

Verified Lisp implementations

verified **LISP** *on*
ARM, x86, PowerPC

Two projects meet

My theorem prover is written in Lisp.
Can I try your verified Lisp interpreter?

Jared Davis

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No, but it could ...

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Running Milawa

Milawa

verified **LISP** *on*
ARM, x86, PowerPC
(TPHOLs 2009)

Running Milawa

Milawa's bootstrap proof:

Milawa

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Milawa's bootstrap proof:

- 4 gigabyte proof file:
  >500 million unique conses

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hopelessly "toy"

Running Milawa

Milawa

Jitawa: *verified* **LISP**
with JIT compiler

Milawa's bootstrap proof:

- 4 gigabyte proof file:
  >500 million unique conses
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Contribution:

- a new verified Lisp which is able to host the Milawa thm prover

Outline

Part 1: **Milawa**

Part 2: **Its new verified runtime**

Part 3: **Mini-demos, measurements**

work by Jared Davis

A short introdution to Milawa

- Milawa is styled after theorem provers such as NQTHM and ACL2,
- has a small trusted logical kernel similar to LCF-style provers,
- ... but does not suffer the performance hit of LCF's fully expansive approach.

work by Jared Davis

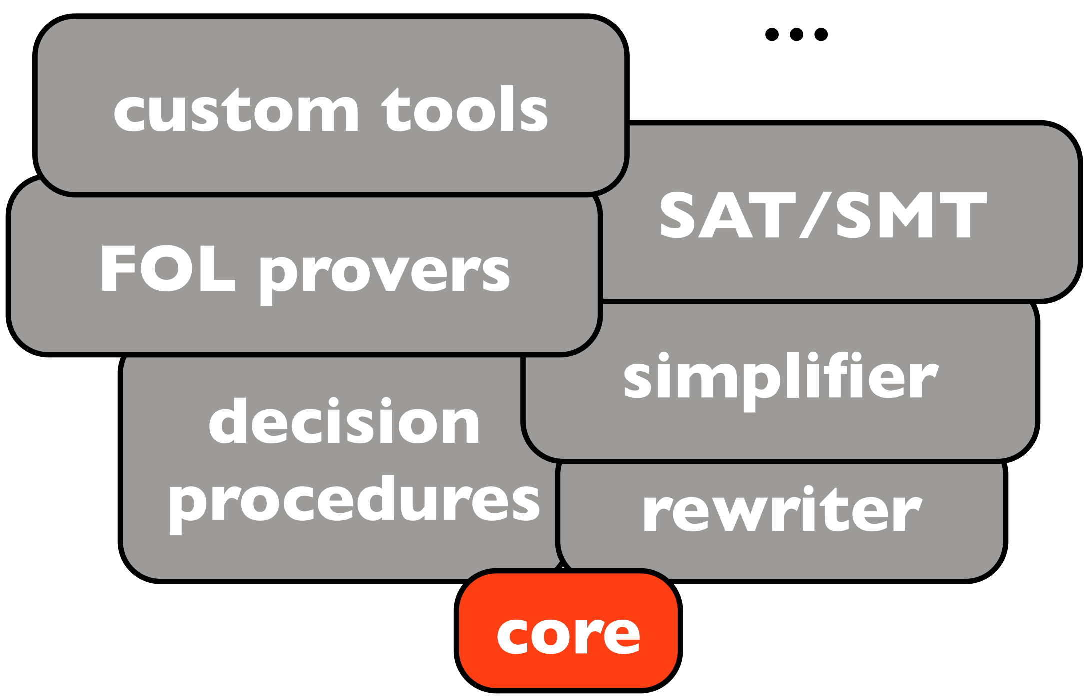
Comparison with LCF approach

LCF-style approach

- all proofs pass through the core's primitive inferences
- extensions steer the core

work by Jared Davis

Comparison with LCF approach



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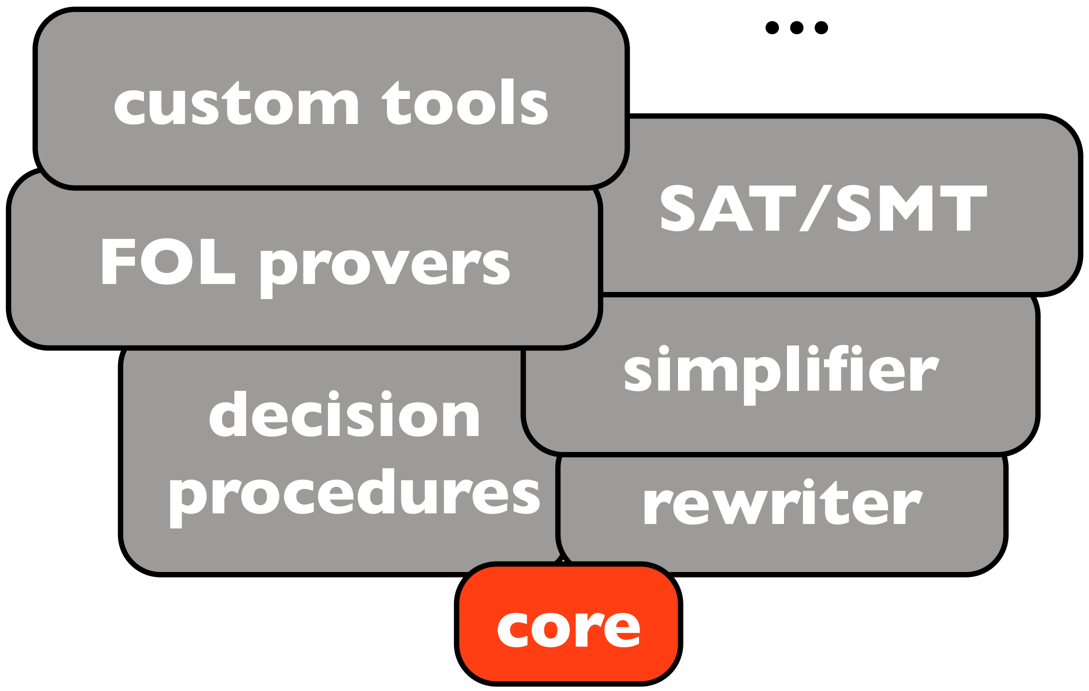

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the Milawa approach

- all proofs must pass the core
- the core can be reflectively extended at runtime

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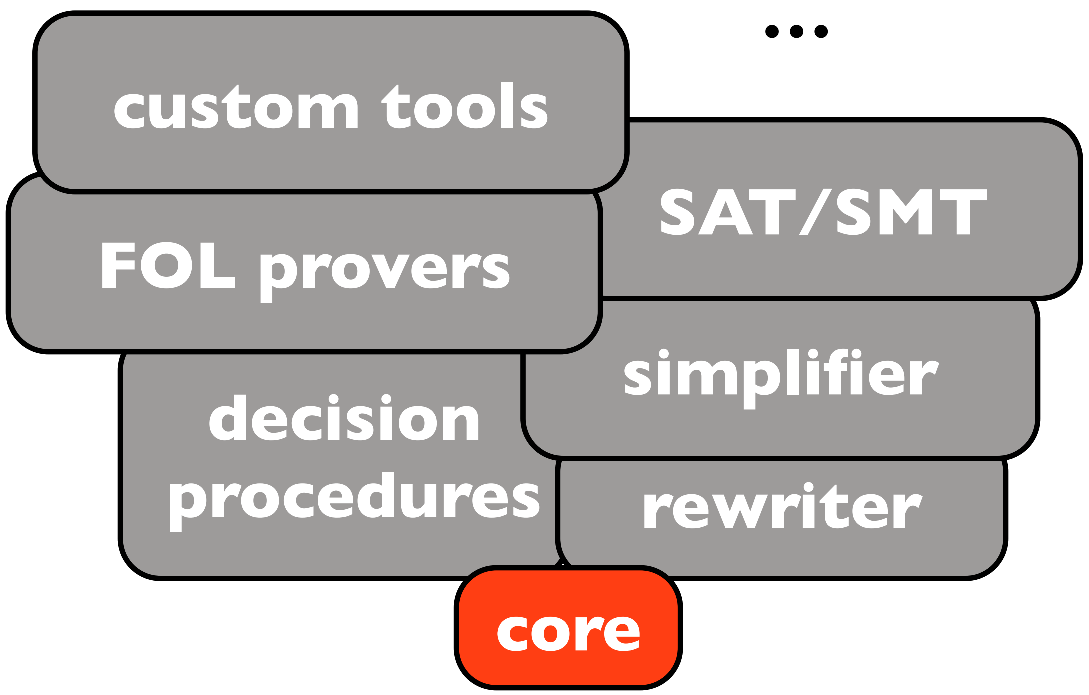

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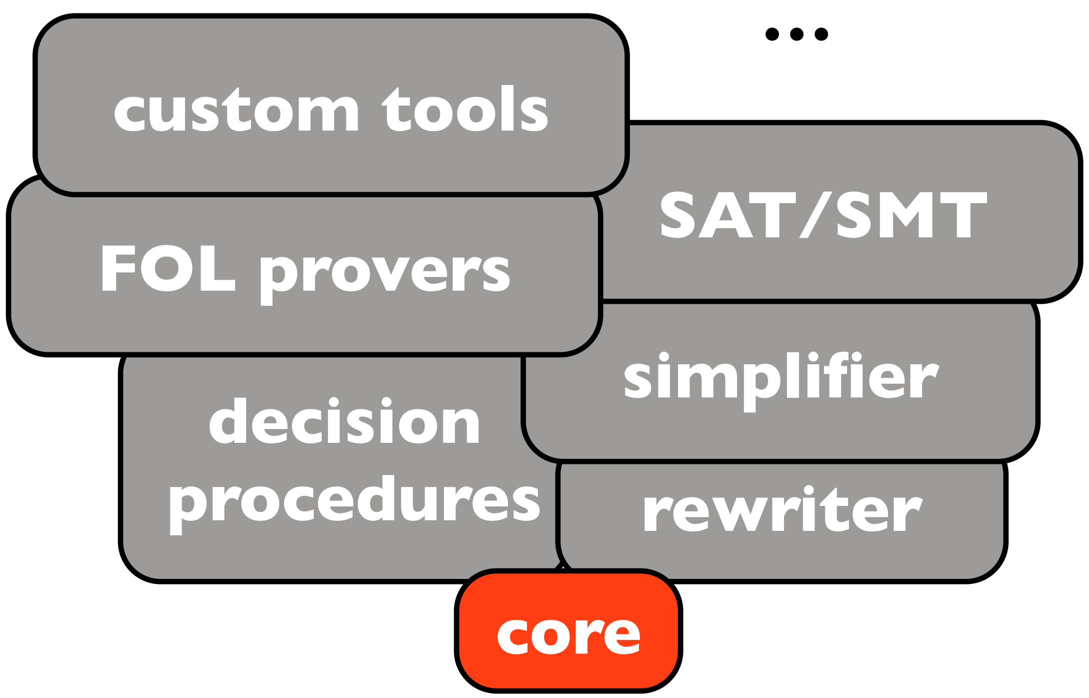

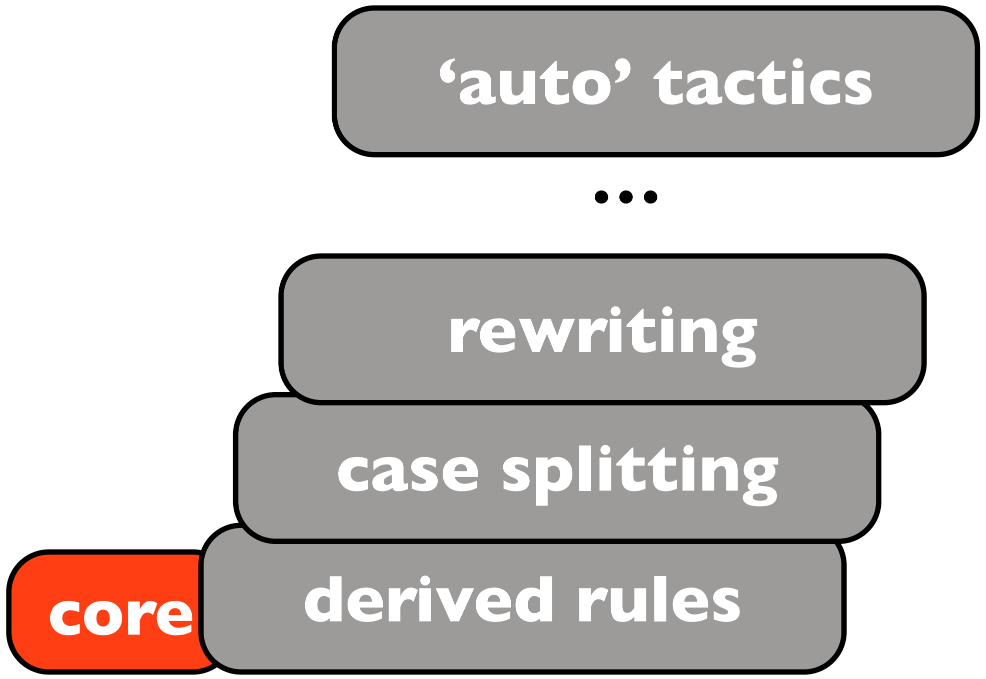

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work by Jared Davis

Bootstrapping Milawa

Output from Milawa's bootstrap proof:

```
(PRINT (1 VERIFY THEOREM-SUBSTITUTE-INTO-NOT-PEQUAL))
(PRINT (2 VERIFY THEOREM-NOT-T-OR-NOT-NIL))
(PRINT (3 DEFINE NOT))
(PRINT (4 VERIFY NOT))
(PRINT (5 DEFINE IFF))
(PRINT (6 VERIFY IFF))
(PRINT (7 VERIFY THEOREM-COMMUTATIVITY-OF-PEQUAL))
...
(PRINT (4611 VERIFY |INSTALL-NEW-PROOFP-LEVEL2.PROOFP|))
(PRINT (4612 SWITCH |LEVEL2.PROOFP|))
(PRINT (4613 VERIFY |BUST-UP-LOGIC.FUNCTION-ARGS-EXPENSIVE|))
...
(PRINT (15685 VERIFY |INSTALL-NEW-PROOFP-LEVEL11.PROOFP|))
(PRINT (15686 SWITCH |LEVEL11.PROOFP|))
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work by Jared Davis

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Output from Milawa's bootstrap proof:

starts with very basic definitions and lemmas

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this event switches to a new extended core

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10 core extensions during bootstrap

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Milawa’s core extensions

work by Jared Davis

Milawa's core extensions

core

can only process primitive inferences, axioms

work by Jared Davis

Milawa's core extensions

supports high-level tactics, similar to ACL2

level 11	Induction and other tactics
level 10	Conditional rewriting
level 9	Evaluation and unconditional rewriting
level 8	Audit trails (in prep for rewriting)
level 7	Case splitting
level 6	Factoring, splitting help
level 5	Assumptions and clauses
level 4	Miscellaneous ground work
level 3	Rules about primitive functions
level 2	Propositional reasoning
core	Primitive proof checker

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Soundness preserved:

each core extension is proved correct w.r.t. the current core before a switch event is allowed.

Milawa's logic

work by Jared Davis

Prop. Schema $\dfrac{}{\neg A \lor A}$

Contraction $\dfrac{A \lor A}{A}$

Expansion $\dfrac{A}{B \lor A}$

Associativity $\dfrac{A \lor (B \lor C)}{(A \lor B) \lor C}$

Cut $\dfrac{A \lor B \quad \neg A \lor C}{B \lor C}$

Instantiation $\dfrac{A}{A/\sigma}$

Induction

Reflexivity Axiom

$x = x$

Equality Axiom

$x_1 = y_1 \rightarrow x_2 = y_2 \rightarrow x_1 = x_2 \rightarrow y_1 = y_2$

Referential Transparency

$x_1 = y_1 \rightarrow \ldots \rightarrow x_n = y_n \rightarrow f(x_1, \ldots, x_n) = f(y_1, \ldots, y_n)$

Beta Reduction

$((\lambda\, x_1 \ldots x_n\, .\, \beta)\; t_1 \ldots t_n) = \beta/[x_1 \leftarrow t_1, \ldots, x_n \leftarrow t_n]$

Base Evaluation

e.g., $1+2 = 3$

Lisp Axioms

e.g., $consp(cons(x, y)) = t$

Sho67, KM98

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w.r.t. ordinals up to ε_0

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evaluation of any lisp primitive applied to constants

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56 axioms describing properties of Lisp primitives

Sho67, KM98

work by Jared Davis

Trusting Milawa

Milawa is trustworthy if:

- logic is sound
- core implements the logic correctly
- runtime executes the core correctly

If the above are proved, then Milawa could be "the most trustworthy theorem prover".

Outline

Part 1: **Milawa**

Part 2: **Its new verified runtime**

Part 3: **Mini-demos, measurements**

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work by Jared Davis

Requirements on runtime

Milawa uses a subset of Common Lisp which

is for most part first-order pure functions over natural numbers, symbols and conses,

uses primitives: `cons car cdr consp natp symbolp equal + - < symbol-< if`

macros: `or and list let let* cond first second third fourth fifth`

and a simple form of lambda-applications.

(Lisp subset defined on later slide.)

work by Jared Davis

Requirements on runtime

...but Milawa also

- uses destructive updates, hash tables
- prints status messages, timing data
- uses Common Lisp's checkpoints
- forces function compilation
- makes dynamic function calls
- can produce runtime errors

(Lisp subset defined on later slide.)

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not necessary

runtime must support

(Lisp subset defined on later slide.)

Runtime must scale

Designed to scale:

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  - must cope with 4 gigabyte input

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Designed to scale:

- dynamic compilation
  - functions compile to native code
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  - space for 2^{31} (2 billion) cons cells (16 GB)
- efficient scannerless parsing + abbreviations
  - must cope with 4 gigabyte input
- graceful exits in all circumstances
  - allowed to run out of space, but must report it

Constructing the new runtime

Conventional method?

1. write simple code
2. then prove it correct

Method used:

1. write approximately correct algorithm implementation
2. start a verification proof
3. iteratively tweak the code and the proof

Constructing the new runtime

Step-by-step:

1. specified input language: syntax & semantics
2. verified necessary algorithms, e.g.
   - compilation from source to bytecode
   - parsing and printing of s-expressions
   - copying garbage collection
3. proved refinements from algorithms to x86 code
4. plugged together to form read-eval-print loop

AST of input language

$$
\begin{array}{lcll}
term & ::= & \textsf{Const}\ sexp & \\
 & | & \textsf{Var}\ string & \\
 & | & \textsf{App}\ func\ (term\ \textsf{list}) & \\
 & | & \textsf{If}\ term\ term\ term & \\
 & | & \textsf{LambdaApp}\ (string\ \textsf{list})\ term\ (term\ \textsf{list}) & \\
 & | & \textsf{Or}\ (term\ \textsf{list}) & \\
 & | & \textsf{And}\ (term\ \textsf{list}) & \text{(macro)} \\
 & | & \textsf{List}\ (term\ \textsf{list}) & \text{(macro)} \\
 & | & \textsf{Let}\ ((string \times term)\ \textsf{list})\ term & \text{(macro)} \\
 & | & \textsf{LetStar}\ ((string \times term)\ \textsf{list})\ term & \text{(macro)} \\
 & | & \textsf{Cond}\ ((term \times term)\ \textsf{list}) & \text{(macro)} \\
 & | & \textsf{First}\ term \mid \textsf{Second}\ term \mid \textsf{Third}\ term & \text{(macro)} \\
 & | & \textsf{Fourth}\ term \mid \textsf{Fifth}\ term & \text{(macro)}
\end{array}
$$

$$
\begin{array}{lcl}
sexp & ::= & \textsf{Val}\ num \\
 & | & \textsf{Sym}\ string \\
 & | & \textsf{Dot}\ sexp\ sexp
\end{array}
$$

$$
\begin{array}{lcl}
func & ::= & \textsf{Define} \mid \textsf{Print} \mid \textsf{Error} \mid \textsf{Funcall} \\
 & | & \textsf{PrimitiveFun}\ primitive \mid \textsf{Fun}\ string
\end{array}
$$

$$
\begin{array}{lcl}
primitive & ::= & \textsf{Equal} \mid \textsf{Symbolp} \mid \textsf{SymbolLess} \\
 & | & \textsf{Consp} \mid \textsf{Cons} \mid \textsf{Car} \mid \textsf{Cdr} \mid \\
 & | & \textsf{Natp} \mid \textsf{Add} \mid \textsf{Sub} \mid \textsf{Less}
\end{array}
$$

AST of input language

Example of semantics for macros:

$$\frac{(\textsf{App}\ (\textsf{PrimitiveFun Car})\ [x], env, k, io) \xrightarrow{\textsf{ev}} (ans, env', k', io')}{(\textsf{First}\ x, env, k, io) \xrightarrow{\textsf{ev}} (ans, env', k', io')}$$

$$
\begin{array}{lcll}
 & \mid & \textsf{List}\ (term\ \textsf{list}) & (\text{macro}) \\
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\end{array}
$$

compile: AST $\longrightarrow$ bytecode list

bytecode	::=	Pop	pop one stack element
	|	PopN *num*	pop n stack elements
	|	PushVal *num*	push a constant number
	|	PushSym *string*	push a constant symbol
	|	LookupConst *num*	push the nth constant from system state
	|	Load *num*	push the nth stack element
	|	Store *num*	overwrite the nth stack element
	|	DataOp *primitive*	add, subtract, car, cons, ...
	|	Jump *num*	jump to program point n
	|	JumpIfNil *num*	conditionally jump to n
	|	DynamicJump	jump to location given by stack top
	|	Call *num*	static function call (faster)
	|	DynamicCall	dynamic function call (slower)
	|	Return	return to calling function
	|	Fail	signal a runtime error
	|	Print	print an object to stdout
	|	Compile	compile a function definition

How do we get compilation to x86?

We have verified compilation algorithm:

compile: AST → bytecode list

but compiler must produce real x86 code....

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Solution:

- bytecode is represented by numbers in memory that are x86 machine code
- we prove that jumping to the memory location of the bytecode executes it

How do we get compilation to x86?

Treating code as data:

$$\forall p\ c\ q.\quad \{p\}\ c\ \{q\}\ =\ \{p * \textsf{code}\ c\}\ \emptyset\ \{q * \textsf{code}\ c\}$$

(POPL'10)

Solution:

- bytecode is represented by numbers in memory that <u>are</u> x86 machine code
- we prove that jumping to the memory location of the bytecode executes it

I/O and efficient parsing

Jitawa implements a read-eval-print loop:

Use of external C routines adds assumptions to proof:

- reading next string from stdin
- printing null-terminated string to stdout

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An efficient s-expression parser (and printer) is proved, which deals with abbreviations:

```
(append (cons (cons a b) c)
        (cons (cons a b) c))
```

```
(append #1=(cons (cons a b) c)
        #1#)
```

Read-eval-print loop

- Result of reading lazily, writing eagerly
- Eval = compile then jump-to-compiled-code
- Specification: read-eval-print until end of input

Correctness theorem

Top-level correctness theorem:

$$\{\,\textsf{init\_state}\ io * \textsf{pc}\ p * \langle \textsf{terminates\_for}\ io \rangle\,\}$$
$$p : \textsf{code\_for\_entire\_jitawa\_implementation}$$
$$\{\,\textsf{error\_message} \vee \exists io'.\ \langle ([], io) \xrightarrow{\textsf{exec}} io' \rangle * \textsf{final\_state}\ io'\,\}$$

Correctness theorem

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list of numbers

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Verified code

```
$ cat verified_code.s

    /*  Machine code automatically extracted from a HOL4 theorem.  */
    /*  The code consists of 7423 instructions (31840 bytes).      */

     .byte   0x48, 0x8B, 0x5F, 0x18
     .byte   0x4C, 0x8B, 0x7F, 0x10
     .byte   0x48, 0x8B, 0x47, 0x20
     .byte   0x48, 0x8B, 0x4F, 0x28
     .byte   0x48, 0x8B, 0x57, 0x08
     .byte   0x48, 0x8B, 0x37
     .byte   0x4C, 0x8B, 0x47, 0x60
     .byte   0x4C, 0x8B, 0x4F, 0x68
     .byte   0x4C, 0x8B, 0x57, 0x58
     .byte   0x48, 0x01, 0xC1
     .byte   0xC7, 0x00, 0x04, 0x4E, 0x49, 0x4C
     .byte   0x48, 0x83, 0xC0, 0x04
     .byte   0xC7, 0x00, 0x02, 0x54, 0x06, 0x51
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How is this verified binary produced?

Demo: proof-producing synthesis (TPHOLs'09)

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Outline

Part 1: **Milawa**

Part 2: **Its new verified runtime**

Part 3: **Mini-demos, measurements**

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Parsing the 4 gigabyte input:

CCL	716 seconds	(9x slower than Jitawa!)
Jitawa	79 seconds	

Quirky behaviour

DEMO

Jitawa mimics an interpreter's behaviour

- to hide the fact that compilation occurs
- to keep semantics as simple as possible
- to facilitate future work (e.g. verify Milawa's core)

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Consequences:

- compiler must turn undefined functions, bad arity and unknown variables into runtime checks/fails.
- mutual recursion is free!

Conclusions

Summary

- new verified runtime
- implements clean Lisp language
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Next year?

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Questions?